

Masses of Four Missing Hadrons having Spin 3/2h Predicted in SU(4) by Standard Model

Imran Khan

*Department of Physics, University of Science and Technology,
Bannu 28100, Pakistan*

Email address: immarwat@yahoo.com

Abstract—In particle physics, study of the symmetry and its breaking play very important role in order to get useful information about the nature. The classification and arrangements of subatomic particles is also necessary to study particle physics. Particles which are building blocks of nature are quarks, gluons and leptons. Baryons and Mesons composed of quarks were arranged by Gell-Mann and Okubo in their well-known Eight-Fold way up to SU(3) symmetry. Standard model of particles is composed of these particles. Particles in SU(4) also make some beautiful patron. These make some multiplets. but all the baryons with spin $J^P = 3/2^+$ in these multiplets have not been observed till date. In this paper the SU(4) multiplets have been organized and studied in an easy and new way. As a result we obtained some clues about the masses and other characteristics of the unknown hyperons. These approximations about the characteristics of the unidentified baryons have been recorded in this article. Mass formula for the baryon SU(4) multiplets have been obtained.

Keywords—Baryons, SU(3), SU(4), Hyperons, Standard Model, Mass Formula.

I. INTRODUCTION

Baryons are made of three quarks (qqq). The three flavors up u , down d , and strange s , imply an approximate flavor SU(3), which requires that baryons made of these quarks belong to the multiplets on the right side of the ‘equation’ $3 \otimes 3 \otimes 3 = 10_S \oplus 8_M \oplus 8_M \oplus 1_A$. Here the subscripts indicate symmetric, mixed symmetry, or anti-symmetric states under interchange of any two quarks [1]. These were classified and arranged by Gell-Mann and Okubo in De-Couplets ($J = 3/2, l = 0$) and Octets ($J = 1/2, l = 0$) with +1 parities [2–4]. The Gell-Mann / Okubo mass formula which relates the masses of members of the baryon octet is given by; [2–4]

$$2(m_N + m_{\Xi}) = 3m_{\Lambda} + m_{\Sigma} \quad (1)$$

While mass formula for De-couplets consists of equal spacing between the rows. The spaces are equal to an average value $-151 \text{MeV}/c^2$.

$$M_{\Lambda} - M_{\Sigma^*} = M_{\Sigma^*} - M_{\Xi^*} = M_{\Xi^*} - M_{\Omega} \quad (2)$$

2. SU(4) MULTIPLETS

Now let's move towards baryon types made from the combination of four quarks, i.e. up u , down d , strange s and charm c . These belong to $SU(4)$ multiplets. The $SU(4)$ multiplets numerology is given by

$$4 \otimes 4 \otimes 4 = 20_S \oplus 20_M \oplus 20_{\bar{M}} \oplus 4_A \quad [5].$$

We are interested in twenty particles having spin $3/2$ and even parity $+1$ forming one of the $SU(4)$ multiplets. These particles are in their ground states, that is with $l = 0$. Their masses and other properties are listed in the Table 1, and shown in figure 1.

Table 1. List of the spin $J^P = 3/2^+$ particles formed from three quarks combinations (qqq) of four quarks u, d, s and c [5].

S.#	Particle Name	Symbol	Quark contents	Rest Mass (MeV/c^2)
1	Delta $^{++}$	Δ^{++} (1232)	uuu	1232 ± 2
2	Delta Plus	Δ^+ (1232)	uud	1232 ± 2
3	Delta Zero	Δ^0 (1232)	udd	1232 ± 2
4	Delta Minus	Δ^- (1232)	ddd	1232 ± 2
5	Sigma Plus	Σ^{*+} (1385)	uus	1382.8 ± 0.4
6	Sigma Zero	Σ^{*0} (1385)	uds	1383.7 ± 1.0
7	Sigma Minus	Σ^{*-} (1385)	dds	1387.2 ± 0.5
8	charmed Sigma $^{++}$	Σ_c^{*++} (2520)	uuc	2517.9 ± 0.6
9	charmed Sigma $^+$	Σ_c^{*+} (2520)	udc	2517.5 ± 2.3
10	charmed Sigma Zero	Σ_c^{*0} (2520)	ddc	2518.8 ± 0.6
11	Xi Zero	Ξ^{*0} (1530)	uss	1531.80 ± 0.32
12	Xi Minus	Ξ^{*-} (1530)	dss	1535.0 ± 0.6
13	charmed Xi $^+$	Ξ_c^{*+} (2645)	usc	$2645.9^{+0.5}_{-0.6}$
14	charmed Xi Zero	Ξ_c^{*0} (2645)	dsc	2645.9 ± 0.5
15	Omega Minus	Ω^- (1672)	sss	1672.45 ± 0.29
16	charmed Omega Zero	Ω_c^{*0} (2770)	ssc	2765.9 ± 2.0
17	double charmed Xi $^{++}$	Ξ_{cc}^{*++}	ucc	Unknown
18	double charmed Xi $^+$	Ξ_{cc}^{*+}	dcc	Unknown
19	double charmed Omega $^+$	Ω_{cc}^{*+}	scc	Unknown
20	triple charmed Omega $^{++}$	Ω_{ccc}^{*++}	ccc	Unknown

Since the mass splitting due to strangeness is typically of order $151 MeV/c^2$ in the de-couplet, which is an effect of about 13% of the masses it contains. The splitting due to charm in the $SU(4)$

particles multiplets is expected to be larger [6]. The ratio $\frac{m_2}{m_1} = \frac{m_4}{m_3}$ indicates the direction of the increasing mass in the Strangeness, or Charm-ness planes in $SU(3)$ and $SU(4)$ multiplets respectively. It gives the mass symmetry breaking ratio for the multiplet [6]. In the $SU(3)$ framework the Gell-Mann / Okubo relation for the $J^P = 1/2^+$ Octets, Eq. (1) and the equal spacing rule for the $J^P = 3/2^+$ de-couplets, Eq. (2) work so nicely that we cannot abandon linear mass formulae for baryons [6]. Using the above relation for baryons of $J^P = 3/2^+$ de-couplet of $SU(3)$ gives a mass symmetry breaking ratio of order 1.0.

II. METHOD

There are four particles in the Table 1, whose masses are unknown. Let's try to make some predictions about the unknown masses of the $\Xi_{cc}^{*++}, \Xi_{cc}^{*+}, \Omega_{cc}^{*+}$, and Ω_{ccc}^{*++} hyperons. The particles listed in Table 1 are grouped in pyramid shape of increasing charm number from $C = 0$ to $C = 3$ as shown in Fig. 1 [5]. Masses of the particles are expressed in round numbers. The bottom of the 'Pyramid' is the $SU(3)$ de-couplet of $J^P = 3/2^+$ particles at $C = 0$. Average masses written on the right side of the Fig. 1 are discussed later in the text.

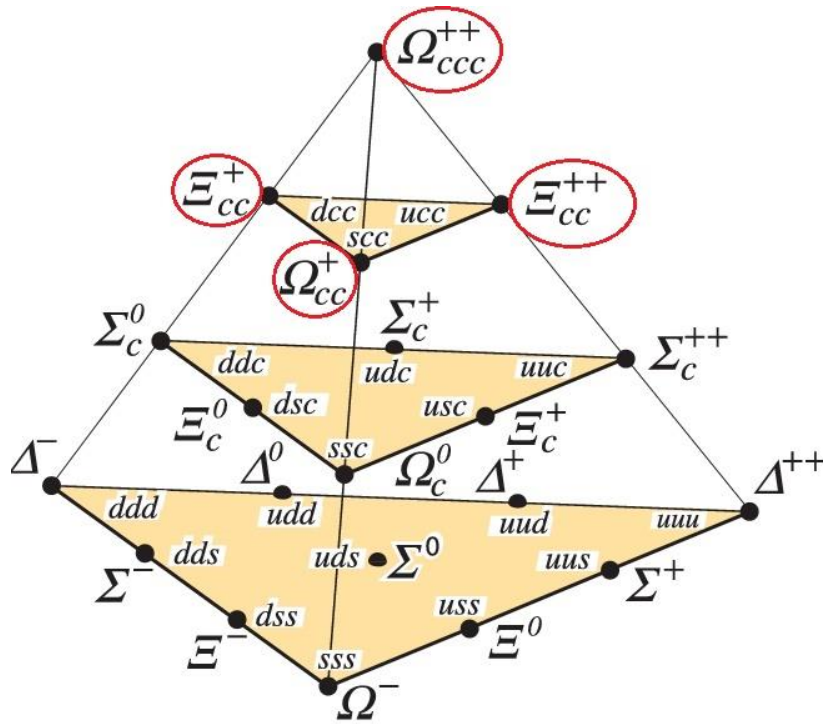


Fig. 1. $SU(4)$ 20-plet of Baryons ($J^P=3/2^+$) made of u, d, s and c quarks, with an $SU(3)$ de-couplet at the bottom [5].

(A) MASS EQUAL SPACING RULE I

Gell-Mann developed this formula. Mass formula for de-couplets consists of equal spacing between the rows. The spaces are equal to an average value $- 151\text{MeV}$.

$$M_{\Delta} - M_{\Sigma^*} = M_{\Sigma^*} - M_{\Xi^*} = M_{\Xi^*} - M_{\Omega} \quad (3)$$

or

$$M_{\Sigma^*} - M_{\Delta} = M_{\Xi^*} - M_{\Sigma^*} = M_{\Omega} - M_{\Xi^*} = 151$$

$$\text{or} \quad \frac{M_{\Sigma^*} - M_{\Delta}}{M_{\Xi^*} - M_{\Sigma^*}} = \frac{M_{\Omega} - M_{\Xi^*}}{151 \text{ MeV}} \quad (4)$$

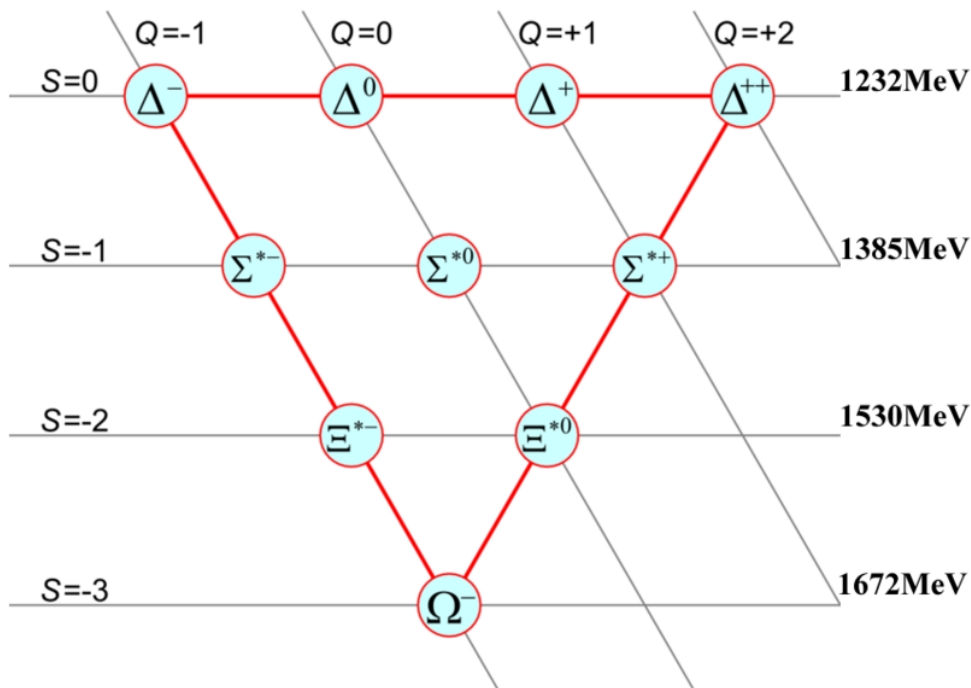


Fig. 2. $SU(3)$ de-couplet of Baryons ($J^P=3/2^+$) made of $u, d,$ and s quarks, used by Gell Mann [5].

Gell-Mann used this formula and predicted the mass of the Ω^- baryon in 1962, equal to $M_{\Omega} = 1685 \text{ MeV}$ [2]. Whereas actual mass of the Ω^- hyperon is equal to 1672 MeV , observed in 1964 [4]. Their mass difference is only 0.72% , or in other words it was 99% true guess.

(B) MASS EQUAL SPACING RULE II

I have developed another rule of equal spacing in layers of the particles with same charge number. According to the charge distribution, average masses of the charge layers are given by;

The twenty particles having spin 3/2 and even parity +1 forming one of the $SU(4)$ multiplets have been studied in reference [6] by using a lengthy complex method. Now we are interested to study the same twenty particles having spin 3/2 and even parity +1 using a very simple and interesting method to extract masses of the unknown particles predicted by standard model of particle physics. These particles are in their ground states, with $l = 0$. For simplicity, natural unit of the mass is used in this article at many places, that is MeV, instead of MeV/c^2 .

Table 2: Mass difference between masses of hyperons having charmness 1 and zero.

S.#	Particle with c	Particle with s	c Particle's mass (MeV/c^2)	s Particle's mass (MeV/c^2)	Mass Difference (MeV/c^2)
1	$\Sigma_c^{*++} (uuc)$	$\Sigma^{*+} (uus)$	2518	1383	1135
2	$\Xi_c^{*+} (usc)$	$\Xi^{*0} (uss)$	2646	1532	1114
3	$\Xi_c^{*0} (dsc)$	$\Xi^{*-} (dss)$	2646	1535	1111
4	$\Omega_c^{*0} (ssc)$	$\Omega^{-} (sss)$	2766	1672	1094

Mass differences between masses of hyperons having charmness equal to 1 and zero are calculated using above stated technique and presented in table 2. Other quark contents of both compared particles are same. Only difference is of s-quark and c-quark. It is found that average mass difference is equal to 1114, ranging from 1094 to 1135 MeV/c^2 . Actual mass difference between these quark masses is equal to $1270-93.4= 1176.6 \text{ MeV}/c^2$.

Table 3: Masses of missing hyperons having two charm quarks are calculated via adding mass differences into mass of particles with one charm quark.

S.#	Particle with cc	Particle with c	Particle's mass Method	Particle's mass (MeV/c^2)
1	$\Xi_{cc}^{*++} (ucc)$	$\Xi_c^{*+} (usc)$	2646+1111	3757±61
2	$\Xi_{cc}^{*+} (dcc)$	$\Xi_c^{*0} (dsc)$	2646+1114	3760±61
3	$\Omega_{cc}^{*+} (scc)$	$\Omega_c^{*0} (ssc)$	2766+1094 2766+1114	3870±62

Then masses of missing hyperons having two charm quarks are calculated via adding mass differences into mass of particles with one charm quark. These are presented in table 3. Thus we have obtained masses of three particles. One particle's Ω_{ccc}^{*++} mass is still not predicted. For this purpose we follow the following procedure.

Gell-Mann/ Okubo for SU(3) de-couplet [8]. Average masses of the particles in layers with charge number -1 and 0, are given by $M_{-1} = 1456 \text{ MeV}$, $M_0 = 2013 \text{ MeV}$ and $M_{+1} = 2568 \text{ MeV}$ respectively. Average mass of the particles in layer with charge number +2, M_{+2} cannot be obtained due to mass of one missing particle.

Difference between average masses of the layers with charge number 0 and -1 is; $M_0 - M_{-1} = 557 \text{ MeV}$. Difference between average masses of the layers with charge number -1 and zero is; $M_{+1} - M_0 = 555 \text{ MeV}$. Adding this value into the average mass of the layer with charge number +1, i.e. $M_{+1} = 2568 \text{ MeV}$ gives; $M_{+1} + 556 \text{ MeV} = 3124 \text{ MeV} = M_{+2}$. Hence we can say that there is an equal spacing rule between these layers, given by;

$$M_2 - M_1 = M_1 - M_0 = M_0 - M_{-1} = 556 \text{ MeV} \quad (6)$$

or

$$\boxed{M_n = M_{n-1} + 556 \text{ MeV} / c^2} \quad (7)$$

Where $n = 0, 1, 2$.

So average mass of the particles in layer with charge number +2, is obtained as equal to $M_{+2} = 3124 \text{ MeV}$.

Now total mass of the layer's particles should be:

$$\text{Total Mass} = 3124 \times 4 = 12496 \text{ MeV} / c^2$$

Mass of the particle is given by:

$$\text{Mass of } \Omega_{ccc}^{++} = 12496 - 3757 - 2518 - 1232 = 4989 \text{ MeV} / c^2$$

$$\text{Mass of } \Omega_{ccc}^{++} = 4989 \pm 70 \text{ MeV} / c^2$$

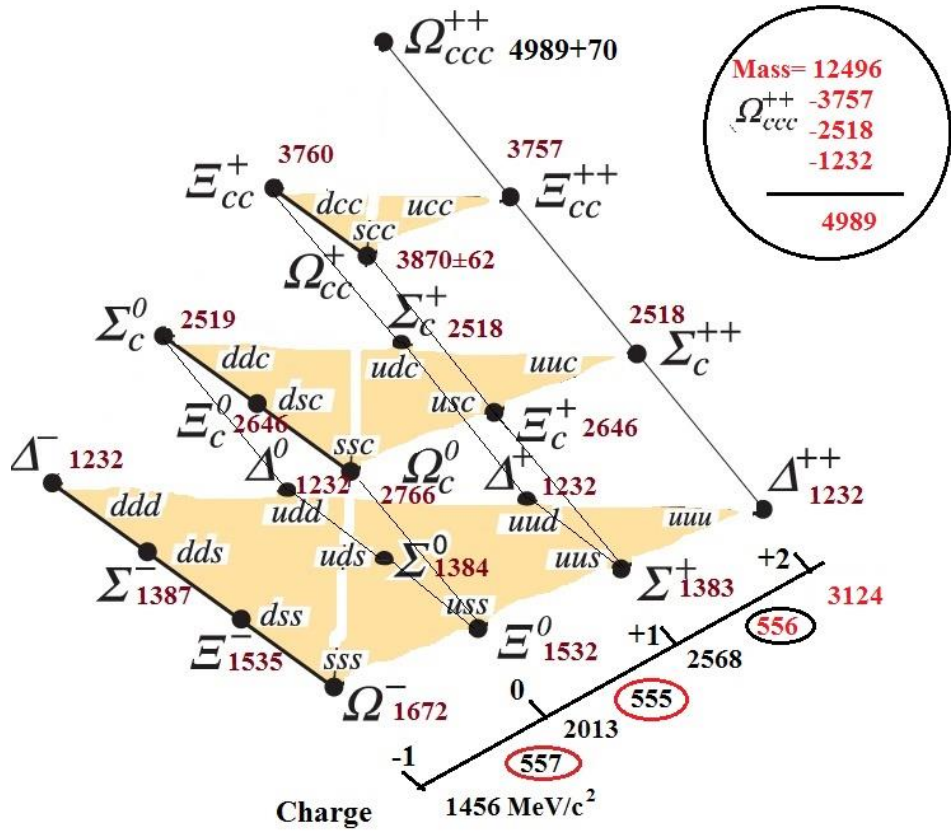


Fig. 5. $SU(4)$ 20-plet of Baryons ($J^P=3/2^+$) with particles masses.

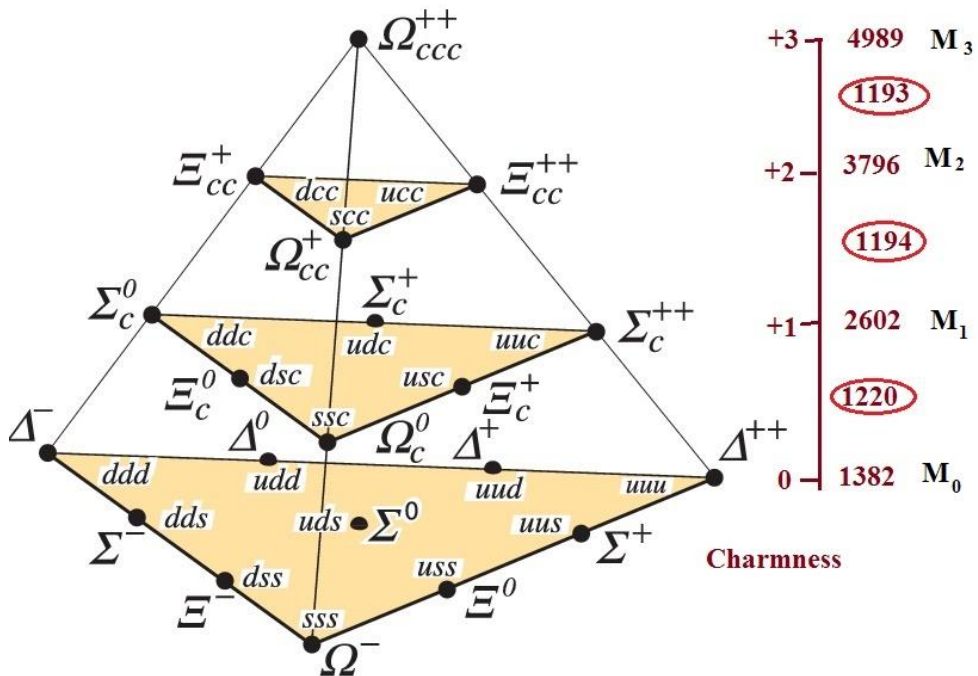


Fig. 6. $SU(4)$ Multiplet, with increasing Mass, Charge number and Charmness from bottom to top.

IV A GENERAL RELATION

The calculations may also be expanded to the whole 20- multiplet, as shown in Figs. 1 & 6, so that a combined formula for all the baryons masses of this multiplet may be extracted.

A general relation between layers of multiplet is given by;

$$M_3 - M_2 = M_2 - M_1 = M_1 - M_0 \approx 1200 \text{ MeV} \quad (8)$$

or

$$\boxed{M_n = M_{n-1} + 1200 \text{ MeV} / c^2} \quad (9)$$

Where n= 1, 2, 3.

And equation for masses of hadrons is given by;

$$\frac{\Xi_{cc}^* + \Xi_{cc}^{*0,-}}{\Omega_{cc}^{*+} + \Sigma} = \frac{2(\Omega_c^{*0} + \Xi_c^{*0} + \Sigma_c + \Omega)}{3(\Delta + \Omega_{ccc}^{*++})} \approx 1$$

Thus the mass of Ω_{ccc}^{*++} particle, estimated using the above method is equal to $4989 \pm 70 \text{ MeV}/c^2$. Statistical Errors are the square root values of the particle masses. Experimentally observed Particle masses may be slightly differ from the above estimated values of masses of all the four particles, due to rough estimates. However in that case the values obtained experimentally should agree with theoretical mass values with in two standard errors. All the predicted masses of the four unknown particles and their other properties are listed in Table 4.

Table 4: Approximated masses and other properties of the unknown particles in Standard Model of $SU(4)$.

S.#	Particles	Particle's mass (MeV/c^2)
1	$\Xi_{cc}^{*++} (ucc)$	3757 ± 61
2	$\Xi_{cc}^{*+} (dcc)$	3760 ± 61
3	$\Omega_{cc}^{*+} (scc)$	3870 ± 62
4	$\Omega_{ccc}^{*++} (ccc)$	4989 ± 70

Now let us check what is the mass difference, $M_2 - M_1 = 1200 \text{ MeV}/c^2$. It is actually mass of the Charm quark, which is the constituent of the particles. Charm number increases by +1 as we go up in the particles pyramid of Fig. 1, and Fig. 6. So this increase in the masses of the particles is due to the mass of the charm quark. Comparing this with the evaluated mass of the charm quark $M_c = 1270 \pm 30$ and $M_c = 1196 \pm 59 \pm 50 \text{ MeV}/c^2$ listed in Ref. [5] gives satisfactory result.

V CONCLUSIONS

$SU(4)$ multiplets of Standard Model have been analyzed in this article and some predictions have been made about the unknown masses and other properties of the Ξ_{cc}^{*++} , Ξ_{cc}^{*+} , Ω_{cc}^{*+} , and Ω_{ccc}^{++} hyperons. Their masses are estimated to be Ξ_{cc}^{*++} (3757 ± 61), Ξ_{cc}^{*+} (3760 ± 61), Ω_{cc}^{*+} (3870 ± 62), and Ω_{ccc}^{++} (4989 ± 70) MeV/c^2 . Statistical errors are the square root values of the estimated masses of particles. Experimentally observed particles' masses may slightly differ from the estimated values of masses of all the four particles, due to rough estimates. However in that case the values obtained experimentally should agree with theoretical mass values with in two standard errors. A general formula for masses has been derived for the $SU(4)$ multiplets, which gives satisfactory results for observed and estimated masses of the particles of $SU(4)$. The mass difference in the multiplets of the $SU(4)$ is due to the mass of Charm quark, and its value is also equal to mass of the charm quark. The results in this research, if proved correct, will be helpful in understanding the particle physics, and will strengthen the Standard Model of particles.

REFERENCES

- [1] Introduction to Elementary Particles, David J. Griffiths, John Wiley & sons, INC, New York, 2008.
- [2] M. Gell-Mann, Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, Switzerland, 1962.
- [3] S. Okubo, "Note on Unitary Symmetry in Strong Interactions", *Progr. Theoret. Phys.* (Kyoto) 27, 949 (1962)
- [4] V. E. Barnes *et.al.*, "Observation of a hyperon with strangeness minus three", *Physical Review Letters*, 12 (8), 204, 1964.
- [5] C. Patrignani *et al.* (Particle Data Group), *Chin. Phys. C*, 40, 100001 (2016)
- [6] R. Aaij *et al.*, "Observation of the doubly charmed baryon Ξ_{cc}^{++} " *PRL* 119, 112001 (2017)

[7] Imran Khan, “Expression for SU(4) multiplet and Masses of undiscovered Baryons in Standard Model”, arxiv.org/abs/1904.05155

[8] Imran Khan, “Approximation of the Masses and other properties of Unknown Hyperons in Standard Model”, <https://arxiv.org/abs/1709.09207>